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RADION PRODUCTION IN e^+e^- COLLISIONS

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Abstract. *We analyse the potential of the e^+e^- collisions to search for the radion in the Randall-Sundrum model. The radion production in the e^+e^- colliders with polarization of e^+, e^- beams was studied in detail. Numerical evaluation shows that if the radion mass is not too heavy then the reaction can give observable cross section at the high energies in future colliders with the high degree of polarization.*

Keywords: radion, electron, section.

I. INTRODUCTION

Randall and Sundrum (RS) have recently presented a simple model[1] which can solve the hierarchy problem by localizing all the standard model (SM) particles on the IR brane. The RS model predicts a new scalar particle, called radion, which stabilize the size of the extra dimension without fine tuning of parameters and is the lowest gravitational excitation in this scenario. The motivation for studying the radion is twofold. Firstly, if radion is the lightest new particle in the RS-type setup then the detection of radion in experiments will be the first important signature of the RS model. In addition, the phenomenological similarity and potential mixing of the radion and Higgs boson warrant detailed study in order to facilitate distinction between the radion and Higgs signals at colliders.

Much research has been done on understanding possible mechanisms for radius stabilization and the phenomenology of the radion field [2-10]. In Refs[11- 12], authors have considered the associated production of radion with Higgs boson at $\gamma\gamma$ colliders. Recently, several authors have also discussed the search of radion in inclusive processes at Tevaron and LHC[13,14]. In our previous works [15, 16], authours have calculated in detail the production cross-sections of radion in external electromagnetic fields and also in the γe^- collisions. In this paper, we will consider the radion production in the high energy collisions with polarization of e^+, e^- beams. The polarization of electron (or positron) beams at the colliders gives a very effective means to control the effect of the SM processes on the experimental analyses. Beam polarization is also an

indispensable tool in identifying and studying new particles and their interactions.
The organization of this paper is follows. In Sec. II, we give a review of the RS model. Section III is devoted to the radion production in high energy e^+e^- colliders. Finally, we summarize our results and make conclusions in Sec. IV.

II. A REVIEW OF RS MODEL

The RS model is based on a 5D spacetime with non-factorizable geometry [1]. The single extradimension is compactified on a S^1/Z_2 orbifold of which two fixed points accommodate two three-branes (4D hyper-surfaces), the Planck brane at $y = 0$ and TeV brane at $y = 1/2$. The ordinary 4D Poincare invariance is shown to be maintained by the following classical solution to the Einstein equation

$$\mathrm{ds}^2=\mathrm{e}^{-2\sigma(\mathrm{y})}\eta_{\mu\nu}\mathrm{dx}^\mu\mathrm{dx}^\nu-\mathrm{b}_0^2\mathrm{dy}^2,\ \sigma(\mathrm{y})=\mathrm{m}_0\mathrm{b}_0|\mathrm{y}|\ , \tag{1}$$

where $x^\mu\,(\mu = 0,1,2,3)$ denote the coordinates on the 4D hyper-surfaces of constant y with metric $\eta_{\mu\nu} = diag(1,-1,-1,-1)$. The m_0 and b_0 are the fundamental mass parameter and compactification radius, respectively.

In the RS model, all the SM fields are confined on the TeV brane. Gravitational fluctuations about the RS metric such as,

$$\eta_{\mu\nu} \longrightarrow \mathrm{g}_{\mu\nu} = \eta_{\mu\nu} + \varepsilon \mathrm{h}_{\mu\nu}(\mathrm{x},\mathrm{y}),\mathrm{b}_0 \to \mathrm{b}_0+\mathrm{b}(\mathrm{x}), \tag{2}$$

yield two kinds of new phenomenological ingredients on the TeV brane: the KK graviton modes $\mathrm{h}^{(\mathrm{n})}_{\mu\nu}(x)$ and the canonically normalized radion field $\phi_0(x)$, respectively defined as

$$\mathrm{h}_{\mu\nu}(\mathrm{x},\mathrm{y}) = \sum_{\mathrm{n}=0}^{\infty}\mathrm{h}^{(\mathrm{n})}_{\mu\nu}(\mathrm{x})\frac{\chi^{(\mathrm{n})}(\mathrm{y})}{\sqrt{\mathrm{b}_0}},\phi_0(\mathrm{x}) = \sqrt{6}\mathrm{M}_\mathrm{P}\Omega_\mathrm{b}(\mathrm{x}), \tag{3}$$

where $\Omega_\mathrm{b}(\mathrm{x}) \equiv \mathrm{e}^{-\mathrm{m}_0[\mathrm{b}_0+\mathrm{b}(\mathrm{x})]/2}$. The 5D Planck mass $M_5(\varepsilon^2 = 16\pi G_5 = \frac{1}{M_5^3})$ is related to its 4D one ($\mathrm{M}_\mathrm{P} = \frac{1}{\sqrt{8\pi G_N}}$) by

$$\frac{\mathrm{M}_\mathrm{P}^2}{2}=\frac{1-\Omega_0^2}{\varepsilon^2\mathrm{m}_0}. \tag{4}$$

Here Ω_0= $\mathrm{e}^{-\mathrm{m}_0\mathrm{b}_0/2}$ is known as the warp factor. Because our TeV brane is arranged to be at $y = 1/2$, a canonically normalized scalar field has the mass multiplied by the warp factor, i.e $m_{phys} = \Omega_0 m_0$. Since the moderate value of $m_0b_0/2 \simeq 35$ can generate TeV scale physical mass, the gauge hierarchy problem is explained.
The interaction of the radion with the SM particles on the visible brance as given in [7,16,19],

$$\mathscr{L} = -\frac{\phi_0}{\Lambda_\phi}\mathrm{T}^\mu_\mu, \tag{5}$$

where $\Lambda_\phi \equiv \sqrt{6}\mathrm{M}_\mathrm{P}\Omega_0$ is the VEV of the radion field. The T^μ_μ is the trace of the energy-momentum tensor of the SM fields localized on the visible brance, which is given at the tree level as

$$T^\mu_\mu = \sum_f m_f\bar{f}f - 2m_W^2W^+_\mu W^{-\mu} - m_Z^2Z_\mu Z^\mu + (2m_{h_0}^2h_0^2 - \partial_\mu h_0\partial^\mu h_0) + \cdots$$

The gravity-scalar mixing arises at the TeV brane by [9]

$$S_{\xi}= -\xi \int d^4x\sqrt{-g_{vis}}R(g_{vis})\widehat{H}^{+}\widehat{H}, \tag{6}$$

where $R(g_{vis})$ is the Ricci scalar for the induced metric on the visible brane or TeV brane, $g_{vis}^{\mu\nu}=\Omega_b^2(x)(\eta^{\mu\nu}+\varepsilon h^{\mu\nu})$, $\widehat{H}$ is the Higgs field before re-scaling, i.e. $H_0 = \Omega_0\widehat{H}$. The parameter ξ denotes the size of the mixing term. With $\xi \neq 0$, there is neither a pure Higgs boson nor pure radion mass eigenstate. This ε term mixes the h_0 and ϕ_0 fields into the mass eigenstates h and ϕ as given by [9, 17]

$$\begin{pmatrix} h_0 \\ \phi_0 \end{pmatrix} = \begin{pmatrix} 1 & -6\xi\gamma/Z \\ 0 & 1/Z \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h \\ \phi \end{pmatrix} = \begin{pmatrix} d & c \\ b & a \end{pmatrix} \begin{pmatrix} h \\ \phi \end{pmatrix}, \tag{7}$$

where

$$\gamma \equiv v_0/\Lambda_\phi, Z^2 \equiv 1-6\xi\gamma^2(1+6\xi) = \beta - 36\xi^2\gamma^2, \beta \equiv 1-6\,\xi\gamma^2, \tag{8a}$$

$$a \equiv \frac{\cos\theta}{Z}, b \equiv -\frac{\sin\theta}{Z}, c \equiv \sin\theta - \frac{6\xi\gamma}{Z}\cos\theta, \tag{8b}$$

$$d \equiv \cos\theta + \frac{6\xi\gamma}{Z}\sin\theta. \tag{8c}$$

The mixing angle θ is defined by

$$\tan 2\theta = 12\gamma\xi Z \frac{m_{h_0}^2}{m_{h_0}^2(Z^2-36\xi^2\gamma^2) - m_{\phi_0}^2} \tag{9}$$

The new fields h and ϕ are mass eigenstates with masses

$$m_{h,\phi}^2 = \frac{1}{2Z^2}\left[m_{\phi_0}^2 + \beta m_{h_0}^2 \pm \sqrt{\left(m_{\phi_0}^2 + \beta m_{h_0}^2\right)^2 - 4Z^2 m_{\phi_0}^2 m_{h_0}^2}\right]. \tag{10}$$

The mixing between the states enable decays of the heavier eigenstate into the lighter eigenstates if kinematically allowed. Overall, the production cross-sections, widths and relative branching fractions can all be affected significantly by the value of the mixing parameter ξ [8, 9, 11]. There are also two algebraic constraints on the value of ξ. One comes from the requirement that the roots of the inverse functions of Eq. (10) are definitely positive. Suggesting that the Higgs boson is heavier, we get

$$\frac{m_h^2}{m_\phi^2} > 1 + \frac{2\beta}{Z^2}\left(1-\frac{Z^2}{\beta}\right) + \frac{2\beta}{Z^2}\left[1-\frac{Z^2}{\beta}\right]^{1/2}. \tag{11}$$

The other one is from the fact that the Z^2 is the coefficient of the radion kinetic term if the coefficient of the radion kinetic term without kinetic mixing. It is therefore required to be positive ($Z^2 > 0$) in order to keep the radion kinetic term definitely positive, i.e

$$-\frac{1}{12}\left(1+\sqrt{1+\frac{4}{\gamma^2}}\right) < \xi < \frac{1}{12}\left(\sqrt{1+\frac{4}{\gamma^2}}-1\right). \tag{12}$$

Note that all phenomenological signatures of the RS model including the radion - Higgs mixing are specified by five parameters as $\Lambda_\phi, \frac{m_0}{M_P}, m_h, m_\phi$, and ξ. In this work, model parameters are chosen as in Refs. [15, 16].

III. RADION PRODUCTION IN e^+e^- COLLISIONS

High energy e^+e^- colliders have been essential instruments to search for the fundamental constituents of matter and their interactions. In our earlier work [16], we have considered the radion production in the high energy γe^- colliders. This section is devoted to the production of radions in the high energy e^+e^- collisions. We now consider the process in which the initial state contains a pair of electron and positron and the final state contains a pair of photon and radion,

$$\mathrm{e}^-(\mathrm{p}_1,\lambda)+\mathrm{e}^+\left(\mathrm{p}_2,\lambda'\right) \to \gamma(\mathrm{k}_1,\tau)+\phi(\mathrm{k}_2) \tag{13}$$

Here p_i, k_i (i = 1,2) stand for the momentum, $\varepsilon_\mu(p_2)$ is the polarization vector of the γ photon and λ, λ' and τ are the helicity of particles, respectively. The amplitude for this process can be written as

$$\mathrm{M}(\mathrm{e}^-\mathrm{e}^+ \to \gamma\phi) = \bar{\mathrm{v}}(\mathrm{p}_2)(\mathrm{ie}\gamma_\nu)\mathrm{u}(\mathrm{p}_1)\left(-\frac{4\mathrm{i}}{\Lambda_\gamma \mathrm{q}^2}\right)[(\mathrm{k}_1.\mathrm{q})\,\mathrm{g}^\nu_\alpha - \mathrm{k}_1^\nu \mathrm{q}_\alpha]\varepsilon^\alpha(\mathrm{k}_1) \tag{14}$$

where $q = p_1 + p_2 = k_1 + k_2$ and $s = (p_1 + p_2)^2$ is the square of the collision energy.
We give some estimates for the cross - section as follows

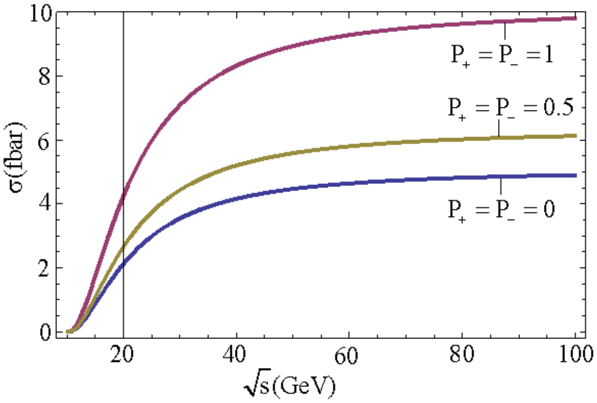

Fig. 1. Cross-section as a function of the collision energy $\sqrt{s}$. The radion mass is taken, $m_\phi = 10$ GeV and typical polarization coefficients are chosen as $P_+ = P_- = 0.0$, 0.5 and 1, respectively.

i) In Fig. 1, we plot the cross-section as a function of the collision energy. We have chosen a relatively low value of the radion mass $m_\phi = 10$ GeV[16] and typical polarization coefficients are $P_+ = P_- = 0.0$, 0.5 and 1, respectively. The figure shows that the cross - section increases when the collision energy $\sqrt{s}$ increases and rises fastly in the region 10 GeV $\leq \sqrt{s} \leq$ 50 GeV, rises gradually when $\sqrt{s} \geq 50$ GeV.

ii) The Fig.2 shows that the cross-section in the polarized coefficients $P_+ = P_- \neq 0$ is larger than that in the polarized coefficients $P_+ = P_- = 0$. The cross-section has the maximum value when both e^+ and e^- are left or right polarized, $\sigma_{max}(P_+ = P_- = \pm 1) \simeq 2\sigma_0(P_+ = P_- = 0)$.

iii) The radion mass dependence of the cross-section σ at fixed high energy, $\sqrt{s} = 2$ TeV[16,23], is shown in Fig.3. The polarized coefficient is chosen as $P_+ = P_- = 0$ and the mass range is 10 GeV $\leq m_\phi \leq$ 200 GeV [16]. From these results, we can see that at the high degree of polarization, the production cross-section of the radion may give observable values at very high energies.

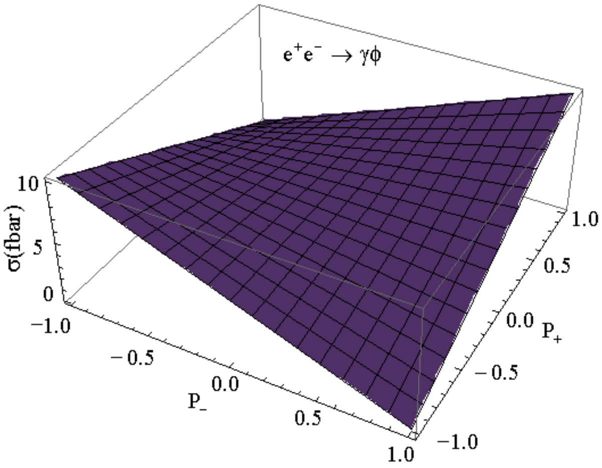

Fig. 2. Cross-section as a function of the polarization coefficients P_- and P_+.

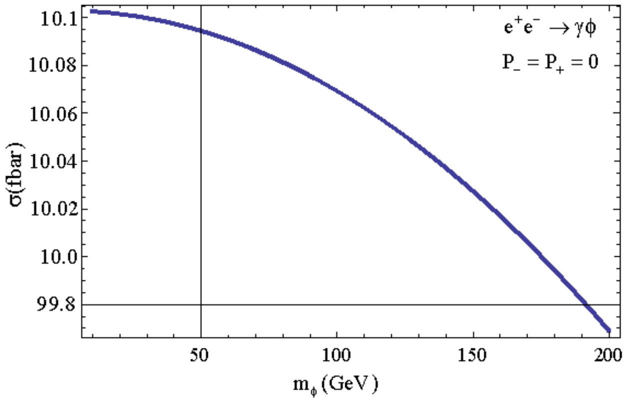

Fig. 3. Cross-section as a function of the radion mass at $\sqrt{s} = 2$ TeV.

IV. CONCLUSION

In this paper, we have evaluated the radion production in e^+e^- collisions. The result show that with the high integrated luminosity and the high polarization of e^+, e^- beams, the production cross-section of radion is much smaller than that in γe^- collisions[16], but it may give observable values at the very high energies in future colliders.

Note that, here we focus only on the case of the radion mass in the range of GeV, which is large enough to avoid radion mediated flavor changing neutral currents [22]. In this mass range, the radion signal at e^+e^- colliders cannot be compared with the high energy colliders at CERN LHC[14], but it is large enough to measure the radion production.

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REFERENCES

[1] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83** (1999) 3370.
[2] C. Csaki, M. Graesser, L. Randall and J. Terning, *Phys. Rev. D* **62** (2000) 045015.
[3] W. D. Goldberge and M. B. Wise, *Phys. Rev. Lett.* **83** (1999) 4922.
[4] W. D. Goldberge and M. B. Wise, *Phys. Lett. B* **475** (2000) 275.
[5] T. Tanaka and X. Montes, *Nucl. Phys. B* **582** (2000) 259.
[6] D. Dominici, B. Grzadkowski, J. F. Gunion and M. Toharia, *Nucl.Phys. B* **671** (2003) 243.
[7] S. Bae, P. Ko, H. S. Lee and J. Lee, *Phys. Lett. B* **487** (2000) 299.
[8] C. Csaki, M. L. Graesser and G. D. Kribs,*Phys. Lett. D* **63** (2001) 065002.
[9] G. F. Giudice, R. Rattazzi and J. D. Wells,*Nucl. Phys. B* **595** (2001) 250.
[10] C. Csaki, J. Hubisz and S. J. Lee, *Phys. Rev. D.* **76** (2007) 125015.
[11] K. Cheung, C. S. Kim and J-H. Song,*Phys. Rev. D.* **67** (2003) 075017.
[12] K. Cheung, C. S. Kim and J-H. Song,*Phys. Rev. D.* **72** (2005) 115015.
[13] H. Davoudiasl, T. McElmurry and A. Soni,*Phys. Rev. D.* **82** (2010) 115028.
[14] V. P. Goncalves and W. K. Sauter,*Phys. Rev. D.* **82** (2010) 056009.
[15] P. V. Dong, H. N. Long, D. V. Soa and N. H. Thao, *JHEP* **1110** (2011) 018.
[16] D. V. Soa, D.T. L. Thuy, N. H. Thao and T. D. Tham, *Mod. Phys. Lett. A* **27**(2) (2012) 1250126.
[17] M. Chaichain, A. Datta, K. Huitu and Z. Vu,*Phys. Lett. B* **524** (2002) 161.
[18] H. Davoudiasl, J. L. Hewett and T. G. Rizzo,*Phys. Rev. D.* **63** (2004) 075004.
[19] C. S. Kim, J. D. Kim ang J-H. Song, *Phys. Rev. D.* **67** (2003) 015001.
[20] I. F. Ginzburg *et al*, *Nucl. Instrum. Meth.* **205** (1983) 47.
[21] V. I. Telnov, *Nucl. Phys. Proc. Suppl.* **82** (2000) 359.
[22] K. Agashe, K. Blum, S. J. Lee and G. Perez, *Phys. Rev. D* **81** (2010) 075012.
[23] S. Bethkea *et al.*, *arXiv:hep-ph/9604332* (1996).